Probability Seminar at Kansai University

Date: Saturday, 28, January 2023

Venue: Kansai University 4th School Area, 3rd Buld., R3401

Program:

<u>13:00 - 14:00</u> Takanobu HARA (Hokkaido University, Japan)

Strong barriers for weighted quasilinear equations

Abstract: In this talk, we construct strong barriers for weighted quasilinear elliptic operators. Two applications are presented: (i) solvability of Poisson-type equations with boundary singular data, and (ii) a geometric version of Hardy inequality.

<u>14:20 - 15:20</u> Kaneharu TSUCHIDA (National Defense Academy of Japan)

On a convergence of positive continuous additive functionals

Abstract: Let $(\mathcal{E}, \mathcal{F})$ be a regular Dirichlet form on $L^2(E; m)$, where E is a locally compact separable metric space and m is a positive Radon measure on E with full support. Let $\mathcal{S}_0$ be the family of all positive Radon measures of finite energy integrals. We introduce a new metric ρ on $\mathcal{S}_0$ by

$$\rho(\mu, \nu) = \sqrt{\mathcal{E}_1(U_1\mu - U_1\nu, U_1\mu - U_1\nu)} \quad \text{for } \mu, \nu \in \mathcal{S}_0,$$

where $U_1\mu$ denotes the 1-potential of μ. Then we can prove that the metric space $(\mathcal{S}_0, \rho)$ is a Polish space, that is, it is complete and separable.

Let A_t (resp. A_t^n) be the positive continuous additive functional (PCAF in short) in the Revuz correspondence with $\mu \in \mathcal{S}_0$ (resp. $\mu_n \in \mathcal{S}_0$). We can show that if $\rho(\mu_n, \mu) \to 0$, there exists a subsequence $\{A_t^{n_k}\}$ of PCAFs $\{A_t^n\}$ such that $A_t^{n_k}$ converges to A_t locally uniformly in $t \geq 0$. Moreover, we give a sufficient condition for holding the above convergence of PCAFs associated with general smooth measures in the case that μ_n and μ are absolutely continuous with respect to m.

This is a joint work with Y. Nishimori, M. Tomisaki and T. Uemura

<u>15:40 - 16:40</u> Panki KIM (Seoul National University, Republic of Korea)

Markov processes with jump kernels degenerating at the boundary

Abstract: In this talk, we discuss Markov processes with jump kernels degenerating at the boundary of the half space. We establish sharp two-sided estimates on the heat kernels of these processes for all admissible values of parameters. The first type of processes we consider are conservative Markov processes on the closed half-space. Depending on the regions where the parameters belong, the heat kernels estimates have three different forms, two of them are qualitatively different from all previously known heat kernel estimates. The second type of processes we consider are the processes above killed either by a critical potential or upon hitting the boundary of the half-space. We establish that their heat kernel estimates have the approximate factorization property with survival probabilities decaying as a power of the distance to the boundary, where the power depends on the constant in the critical potential. Our results are the first sharp two-sided estimates for the heat kernels of non-local operators with jump kernels degenerate at the boundary. By using the heat kernel estimates, we obtain sharp two-sided Green function estimates that cover the main result of Kim-Song-Vondraček, Journal of the European Mathematical Society(JEMS), 2022+. Depending on the regions where parameters belong, the estimates on the Green functions are different. As applications, we completely determine the region of the parameters where the boundary Harnack principle holds or not. This talk is mainly based on a joint work with Soobin Cho, Renming Song and Zoran Vondraček (arXiv:2211.08606 [math.PR])