International Workshop on

Dirichlet Forms and Related Topics

in Honor of

Professor Masatoshi Fukushima's 米寿(Beiju)

Abstracts

August 22-26, 2022

Kansai University

August 22-24: Centenary Memorial Hall

(100 周年記念会館)

August 25-26: Bldg.3, Area 4 Room 3401

(第 4 学舎 3 号館 3401)

New Mathematical View on Quantum Mechanics

Mu-Fa Chen (Jiangsu Normal University and Beijing Normal University)

This talk is an overview of the personal trip of the study: from birth-death processes to quantum mechanics. The first point of the motivation came from the study on the principal eigenvalue of the birth-death processes with killing. The difficulty was overcome in terms of an isospectal transform, by using the harmonic function of the original operator, removing its killing term. The second point of the motivation came from the study of the complex eigenvector of non-symmetrizable Markov chains. This leads to the new object called Hermitizable matrix, as an extension of the symmetrizable ones which we have studied since 1979, closely related to the Dirichlet form theory. It is very unexpected that the spectrum of each of the Hermitizable matrix can be expressed as the union of the spectrums of some birth-death processes. In parallel, the Hermitizable problem for complex second-order differential operators with real domain, as well as for the Schrödinger operators is studied. Anyhow, the story leads to a new framework, new spectral theory, and new algorithms in the study of quantum mechanics.

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Laplacians on loops and connections on graphs

Yves Le Jan (Université Paris-Sud)

We present some results concerning Laplacians on connections and gauge groups on a graph.

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Generators of Lévy processes studied in the setting of distributions and in the setting of Dirichlet forms

Niels Jacob (Swansea University)

We start to study generators of Lévy processes in the setting of distributions in the sense of Laurent Schwartz. As translation invariant operators these operators are of course convolution operators. In particular, we want to understand which results known for general partial differential operators with constant coefficients can be transferred. A surprising fact is that many of the basic notions used when investigating partial differential operators can not even be defined in our setting. In addition, anisotropy, and the lack of smooth of the symbols of our generators lead to much different type of results. For example, already a proper notion of ellipticity causes problems. Adding the assumption that the symbols are real-valued, or more generally satisfy the sector condition, then we may work in the scale of associated Bessel potential spaces, i.e., the domains of the powers of the generator of the associated Dirichlet forms. Here we now encounter the situation that operators which we must expect to behave differently in the setting of distributions, e.g., in calculating the singular support of $q(D)u$, behave as mappings in the scale of these spaces identically under what could be seen as a generalised ellipticity condition. Consequently, one will get identical Nash inequalities for these operators implying identical diagonal decay of the corresponding heat kernels. However, the off-diagonal behaviour will in general be quite different.

We will compare approaches to get off-diagonal bounds for heat kernels by using Dirichlet form techniques, i.e., conditions on the Lévy measure, and by trying to investigate inverse Fourier transforms. In this talk we are more concerned with problems arising when trying to establish a general analytic theory for generators of Lévy processes than in dealing with more special results. For this reason we will make only a few remarks to the case of operators with variables "coefficients".

Two Results on Martingale Representation

Patrick Fitzsimmons (University of California at San Diego)

An old result of F.W. Gehring asserts that if f is an a.e. finite measurable function on the unit circle in the complex plane, then there is a harmonic function h in the unit disk with non-tangential limits equal to f at a.e. point of the circle. It's clear that h is far from unique – think of the Poisson integral u of a singular probability measure on the circle, and consider $h + c \cdot u$, for any constant $c \in \mathbb{R}$.

A conceptually related result, due to R.M. Dudley (but anticipated by C.W. Lamb), concerns a real-valued Brownian motion $\{B_t : 0 \leq t \leq 1\}$, and asserts that if F is a random variable measurable over $\sigma\{B_t : 0 \leq t \leq 1\}$ then there is a predictable process $H = (H_t)_{0 \leq t \leq 1}$ with $\int_0^1 H_t^2 dt < \infty$ a.s. such that $F = \int_0^1 H_1 dB_t$ a.s. Once again, H is not unique. But if F is integrable then we have the martingale $M_t := \mathbb{E}[F|\mathcal{F}_t]$ which admits a stochastic integral representation $M_t = \int_0^t K_s dB_s, 0 \leq t \leq 1$, a.s., and of course $F = M_1 = \int_0^1 K_s dB_s$, a.s. Under certain additional conditions we have $\mathbb{E} \int_0^1 K_s^2 ds \leq \mathbb{E} \int_0^1 H_s^2 dx$, for any process H as in Dudley's stochastic integral representation of F.

My goal in this work is to gain a probabilistic understanding of Gehring's result, and to explore side conditions on H ensuring some sort of uniqueness.

Equivalence of the strong Feller properties of analytic semigroups and associated resolvents

Kazuhiro Kuwae (Fukuoka University)

In this talk, I give sufficient conditions for the equivalence between semigroup strong Feller property and resolvent strong Feller property. This is a joint work with Seiichiro Kusuoka and Kouhei Matsuura.

Silverstein extension and Fukushima extension

Jiangang Ying (Fudan University)

The notion of Fukushima extension is introduced and it is proved that a Dirichlet extension of a Dirichlet form can be decomposed uniquely into a Silverstein extension and a Fukushima extension.

Some known results on Fukushima extension of 1-dim Brownian motion and stable processes are illustrated. While Silverstein extension is constructed by changing only the structure of the form on the boundary, Fukushima extension is obtained by changing essentially the whole structure and much more difficult to describe.

Degenerate non-linear Fokker-Planck equations and corresponding McKean-Vlasov equations: weak uniqueness and Markov property

Michael Röckner (Bielefeld University)

In this talk we shall first review our recent results about the equivalence of non-linear Fokker-Planck equations and McKean Vlasov SDEs (see [1]). Then we shall recall our results on existence of weak solutions to both such equations in the singular case, where the measure dependence of the coefficients are of Nemytskii-type (see [2]). The main new results to be presented are about weak uniqueness of solutions to both nonlinear Fokker-Planck equations and the corresponding McKean-Vlasov SDEs in the case of (possibly) degenerate diffusion coefficients (see [3]). As a consequence of this and [4] one obtains that the laws on path space of the solutions to the McKean-Vlasov SDEs form a nonlinear Markov process in the sense of McKean.

Joint work with:
Viorel Barbu, Al.I. Cuza University and Octav Mayer Institute of Mathematics of Romanian Academy, Iaşi, Romania

References

[1] Barbu/R.: Annals of Probability 2020.

[2] Barbu/R.: J. Funct. Anal. 2021.

[3] Barbu/R.: arXiv:2203.00122v2.

[4] Ren/R./Wang: J. Diff. Equations 2022

Tail estimates of the heat semigroup for jump processes

Alexander Grigor'yan (Bielefeld University)

We consider a Dirichlet form of jump type on a doubling space and prove that, under certain hypotheses, an L^p-tail estimate of the heat semigroup and that of the jump kernel are equivalent. Consequently, we obtain pointwise off-diagonal upper bounds of the heat kernel under integral upper bounds of the jump kernel. This is a joint work with Jiaxin Hu and Eryan Hu.

Conformally Invariant Random Geometry on Riemannian Manifolds of Even Dimension

Karl-Theodor Sturm (University of Bonn)

We construct and study conformally invariant, log-correlated Gaussian random fields on compact Riemannian manifolds of general even dimension uniquely defined through its covariance kernel given as inverse of the Graham-Jenne-Mason-Sparling (GJMS) operator. The corresponding Gaussian Multiplicative Chaos is a generalization to the n-dimensional case of the celebrated Liouville Quantum Gravity measure in dimension two. Finally, we study the Polyakov-Liouville measure on the space of distributions on M induced by the copolyharmonic Gaussian field, providing explicit conditions for its finiteness and computing the conformal anomaly.

Infinite particle systems with hard-core and long-range interaction

Hideki Tanemura (Keio University)

A system of Brownian hard balls is regarded as a reflecting Brownian motion in the configuration space and can be represented by a solution to a Skorohod-type equation. In this talk, we consider the case that there are an infinite number of balls, and the interaction between balls is given by the long-range pair interaction. We discuss the existence and uniqueness of strong solutions to the infinite-dimensional Skorohod equation.

On the continuity of half-plane capacity with respect to Carathéodory convergence

Takuya Murayama (Kyushu University)

Loewner's differential equation describes the growth of planar continua, called hulls, and is used at several places, for example, in SLE theory. The half-plane capacity, a kind of capacity of hulls, plays a role of intrinsic time-parameter for this growth. We show that the halfplane capacity is continuous when hulls vary continuously with respect to the Carathéodory convergence. Moreover, using the theory of Brownian motion with darning (BMD), we extend the results to finitely-connnected planar domains. This is motivated by the recent development on the relation between the Komatu-Loewner equation and BMD.

On heat kernel upper bounds for symmetric Markov semigroups

Jian Wang (Fujian Normal University)

In this talk, we show that under some mild assumptions Nash-type inequalities only can yield off-diagonal heat kernel upper bounds for symmetric Markov processes. This extends the famous

work by Carlen-Kusuoka-Stroock. We also show a new and direct approach to obtain large time heat kernel estimates for symmetric strongly local Dirichlet forms under general volume doubling setting.

Heat kernels for reflected diffusions with jumps on inner uniform domains

Takashi Kumagai (Waseda University)

In this talk, we present some recent progress in the study of sharp two-sided heat kernel estimates for a large class of symmetric reflected diffusions with jumps on the closure of an inner uniform domain in a length metric space. We also discuss generalized Davies' method for the off-diagonal upper bounds of the heat kernel that is used in the proof of the above mentioned estimates. This talk is based on joint work with Zhen-Qing Chen, Panki Kim and Jian Wang.

Stochastic partial differential equations with local monotone coefficients

Tusheng Zhang (University of Manchester)

Considered in a Gelfand triple, the well posedness of stochastic partial differential equations with monotone or particular type of local monotone coefficients is now well understood. In this talk, we will report recent progresses on the well-posedness of stochastic partial differential equations which have fully local monotone coefficients. The results apply to many interesting models/examples. This is a joint work with Michael Röckner and Shijie Shang.

On the upper rate functions of some time inhomogeneous diffusion processes

Daehong Kim (Kumamoto University)

In this talk, we will discuss a condition for the upper escape bound of some time inhomogeneous diffusion process associated with a family of regular and local Dirichlet forms. In particular, by making full use of Gaussian type's heat kernel estimates, we establish integral tests for an upper rate function of the time inhomogeneous diffusion process with a coefficient that is not necessarily bounded concerning space and time. Joint work with Yoichi Oshima.

Estimates of fundamental solutions for critical Schrödinger forms

Masaki Wada (Fukushima University)

Let $\{X_t\}_{t\geq 0}$ be the rotationally invariant α-stable process with generator $-(-\Delta)^{\alpha/2}$ $(0 < \alpha \leq 2)$ on $\mathbb{R}^d$ and denote by $(\mathcal{E}, \mathcal{F})$ the corresponding Dirichlet form. Assume the process $\{X_t\}_{t\geq 0}$ is transient and let $G(x, y)$ be the Green kernel defined by

$$G(x, y) = \int_0^\infty p(t, x, y)dt,$$

where $p(t, x, y)$ is the transition density function of $\{X_t\}_{t\geq 0}$. Then we can define a certain class of positive measures $\mathcal{K}_\infty$ (i.e. Kato class, Green-tight and of finite 0-order energy integral) by using the Green kernel. For $\mu \in \mathcal{K}_\infty$, we consider the Schrödinger form

$$\mathcal{E}^\mu(u, u) = \mathcal{E}(u, u) - \int_{\mathbb{R}^d} u^2(x)\mu(dx)$$

and denote by $p^\mu(t, x, y)$ the corresponding fundamental solution. If μ is small enough in the sense of

$$\inf\left\{\mathcal{E}(u, u) \mid \int_{\mathbb{R}^d} u^2(x)\mu(dx) = 1\right\} > 1,$$

$p^\mu(t, x, y)$ is comparable to $p(t, x, y)$, i.e. for some positive constants c_1 and c_2,

$$c_1 p(t, x, y) \leq p^\mu(t, x, y) \leq c_2 p(t, x, y) \tag{1}$$

and thus μ is said to be subcritical. If μ is critical in the sense of

$$\inf\left\{\mathcal{E}(u, u) \mid \int_{\mathbb{R}^d} u^2(x)\mu(dx) = 1\right\} = 1,$$

$p^\mu(t, x, y)$ has different behavior from (1). In this talk, we consider the large time asymptotics of $\int_{\mathbb{R}^d} p^\mu(t, x, y)f(y)dy$, where f is a positive Borel measurable function.

On diffusions with discontinuous scales

Liping Li (Bielefeld University, Germany and Fudan University, China)

In this talk we will introduce the Dirichlet forms related to a triple (I, s, m), where I is a closed interval, s is a (not necessarily strictly) increasing function on I, and m is a certain positive measure on I. These Dirichlet forms are associated with so-called skip-free Hunt processes on an almost closed set in one dimension, which generalize classical regular diffusions. We will also show the correspondence between Dirichlet forms, skip-free Hunt processes and quasidiffusions.

Local time penalizations with various clocks for Lévy processes

Kouji Yano (Kyoto University)

For one-dimensional Lévy processes, we discuss local time penalizations, a generalization of conditioning to avoid zero. The limits are taken via certain families of random times, called clocks. The limit processes may differ according to the choice of the clocks when the original Lévy process is recurrent and of finite variance. This talk is based on a joint work with Shosei Takeda, arXiv:2203.08428.

Weak Hardy and Poincaré inequalities and criticality theory

Marcel Schmidt (University of Leipzig)

In this talk we discuss weak Hardy and Poincaré inequalities for quadratic forms satisfying the first Beurling-Deny criterion. We employ these inequalities to establish a criticality theory for such forms and to study completeness of the extended form domain.

Potential theoretic analogues of complex analytical theory on infinite graphs

Hiroshi Kaneko (Tokyo University of Science)

Recently several researches on discrete analogues of classical theory in complex analysis such as Baker and Norine's Riemann-Roch theorem on finite graphs have been undertaken. On the other hand, Laine and Tohge presented a tropical Nevanlinna theory, where an analogy of classical Nevanlinna theory was explored for polygonal interpolations on $\mathbb{R}$ of the discrete functions on the straight line graph over $\mathbb{Z}$. By accepting this interpretation, a similar result on general graphs to the analogous lemma of classical logarithmic derivative by Halburd and Southall could be in our focus. We recall that the lemma of logarithmic derivative (abr. LLD) plays a main role of the proof of the second main theorem in classical Nevanlinna theory. The original version of LLD concludes that, for any non-constant meromorphic function f on $\mathbb{C}$,

$$\frac{1}{2\pi}\int_0^{2\pi} \log^+ |\frac{f'}{f}(re^{i\theta})| d\theta \leq o(T(r))$$

as $r \to \infty$ outside an exceptional set, where $T(r)$ stands for the characteristic function of f. The left-hand side is denoted by $m(r, \frac{f'}{f})$ in the classical Nevanlinna theory.

In this talk, extensions of the Riemann-Roch theorem to infinite graphs will be dealt with, where harmonic extensions of functions and chip-firings for divisors to represent "dimension" of a divisor given on infinite graphs play important roles. We start with the complex analytical terminolgies on finite graph $G = (V_G, E_G)$ with the set V_G of vertices and the set E_G of edges in the following table:
In what follows, a divisor D represented by $D = \sum_{x \in V_G} \ell(x) 1_{\{x\}}$ with non-negative valued ℓ is said to be effective and $i_{(G,C)} = \min\{|\sum_{x \in V_G} \ell(x) i(x)| \in (0, \infty) \mid \ell : V_G \to \mathbb{Z}\}$.

notions	uniformly weighted graph	graph with weighted edges
weight on edges	1	conductance $C_{x,y}$ between x and y
meromorphic function	$\mathbb{Z}$-valued function f	
punctured neighourhood of x	$N(x) = \{y \in V_G \mid \{x, y\} \in E_G\}$	
Laplacian $\Delta f(x)$ of f at x	$\sum_{y \in N(x)} (f(x) - f(y))$	$\sum_{y \in N(x)} C_{x,y}(f(x) - f(y))$
weight at vertices	1	$i(x) = \min\{\lvert\Delta f(x)\rvert \mid f : V_G \to \mathbb{Z}$ satisfying $f(x) = 0$ and $\Delta f(x) \neq 0\}$
divisor D	$\sum_{x \in V_G} \ell(x) 1_{\{x\}}$	$\sum_{x \in V_G} \ell(x) i(x) 1_{\{x\}}$ with $\ell : V_G \to \mathbb{Z}$
degree $\deg(D)$ of divisor	$\sum_{x \in V_G} \ell(x)$	$\sum_{x \in V_G} \ell(x) i(x)$
canonical divisor K_G	$\sum_{x \in V_G} \{\#N(x) - 2\} 1_{\{x\}}$	$\sum_{x \in V_G} \{\sum_{y \in N(x)} C_{x,y} - 2i(x)\} 1_{\{x\}}$
Euler characteristic $\mathfrak{e}_{(G,C)}$	$\#V_G - \#E_G$	$\sum_{x \in V_G} i(x) - \sum_{\{x,y\} \in E_G} C_{x,y}$
Riemann-Roch theorem	$r(D) - r(K_G - D) = \deg(D) + \mathfrak{e}_{(G,C)}$	

For any divisor D and non-negative integer k, $E_k(D) = \{\text{effective divisor } E \mid \deg(E) = i_{(G,C)}k$ such that $D - E + \Delta f$ is effective for some $f : V_G \to \mathbb{Z}\}$. We note that $E_0(D)$ is either empty set or the set consisting only of the zero divisor in general. We also define a $\{-i_{(G,C)}, 0, i_{(G,C)}, 2i_{(G,C)}, \dots\}$-valued function r on the set consisting of all divisors by

$$r(D) = \begin{cases} -i_{(G,C)}, & \text{if } E_0(D) = \emptyset, \\ \max\{i_{(G,C)}k & \mid E_k(D) \text{ consists of all effective divisors} \\ & \text{of degree } i_{(G,C)}k\}, \text{ otherwise,} \end{cases}$$

which is called dimension of the divisor D.

We consider a generalization of Laine-Tohge Nevanlinna theory to the case in more general locally finite graphs, relying on Dirichlet space theory. For this purpose, we consider finitely branched tree instead of straight line graph over $\mathbb{Z}$ and obtain a Halburd-Southall type LLD on the tree. From this LLD, in particular, we can show that for any non-constant tropical meromorphic function f of finite order on a tree $T = (V, E)$ determined by set V of vertices and the set E of edges

$$m(r, f_c - f) = o(T(r)) \text{ as } r \to \infty$$

outside an exceptional set under certain conditions on the degrees of the vertices of T and isotropic and growth conditions on f. Here, $m(r, f_c - f)$ is required to be reformulated as a tropical analogue of $m(r, \frac{f'}{f})$ as in existing results on Nevanlinna theory on discrete space. These results are obtained by a collaborative research project with A. Atsuji.

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Quasisymmetric Gaussian uniformization is impossible for Brownian motion on the Sierpiński carpet

Naotaka Kajino (Kyoto University)

It is an established result in the field of analysis of diffusion processes on fractals, that the transition density of the diffusion typically satisfies analogs of Gaussian bounds which involve a

space-time scaling exponent β greater than two and thereby are called SUB-Gaussian bounds. The exponent β, called the walk dimension of the diffusion, could be considered as representing "how close the geometry of the fractal is to being smooth". It has been observed by Kigami in [Math. Ann. **340** (2008), 781–804] that, in the case of the standard two-dimensional Sierpiński gasket, one can decrease this exponent to two (so that Gaussian bounds hold) by suitable changes of the metric and the measure while keeping the associated Dirichlet form (the quadratic energy functional) the same. Then it is natural to ask how general this phenomenon is for diffusions on fractals. In fact, it turns out that the above phenomenon, that one can decrease the exponent β to two so that Gaussian bounds hold, seems to happen only for a very limited class of self-similar fractals. This talk is aimed at presenting the result that this phenomenon indeed does NOT happen for the Brownian motion on the standard (two-dimensional) Sierpiński carpet, as well as for the Brownian motion on the standard three- and higher-dimensional Sierpński gaskets. This talk is based on joint works with Mathav Murugan (UBC). The result for the standard Sierpi 卒 nski carpet is in progress, and that for the standard higher-dimensional Sierpi 卒 nski gaskets is given in arXiv:2008.12836.

Discrete approximations of reflected Brownian motions by Markov chains on partitions of domains

Kouhei Matsuura (University of Tsukuba)

In this talk, we will consider discrete approximation of reflected Brownian motions on smooth domains. Our approximation is a sequence of Markov chains on partitions of the domain, where the partitions allow uneven or random ones. We provide sufficient conditions for the weak convergence of the Markov chains. This talk is based on joint work with Masanori Hino (Kyoto University) and Arata Maki.

Dirichlet form approach to infinite-dimensional Wiener processes with long-granged and strong interactions

Hirofumi Osada (Chubu University)

The planner Gaussian analytic function (planner GAF) is an entire function whose coefficients are independent Gaussian random variables with mean free and unit variance. The zero points of the planner GAF define a random point field (RPF) in the plane with translation and rotation invariance. The random point field is called GAF in this talk. The GAF has a mean rigidity. The interaction of GAF is not a potential type. Thus, GAF is very different from usual RPFs with interaction potentials. I show that GAF does not have any infinitedimensional stochastic differential equation to describe the dynamics. Using the Dirichlet form approach, I construct the dynamics of GAF and investigate its dynamical property of GAF.