# Dirichlet forms and their geometry -program and abstracts-

Date: March 18th (Sat) – March 23rd (Thu), 2017.

Venue: Tohoku University GSIS 2F Lecture Hall

Organizers: J. Masamune (Hokkaido Univ.), N. Obata (Tohoku Univ.)
M. Schmidt (U. Jena, Germany), T. Uemura (Kansai Univ.), M. Wada (Tohoku Univ.)

March 18th, Saturday

08:30– Registration

09:25–10:10 Takashi Shioya
Compactification of the space of metric measure spaces

10:20–11:05 Asuka Takatsu
High-dimensional metric measure limit of Stiefel and Grassmann manifolds

11:15–12:00 Melchior Wirth
Geometry of Dirichlet forms under order isomorphisms

13:20–14:50 Masatoshi Fukushima
Liouville property of harmonic functions of finite energy for Dirichlet forms

15:20–16:50 Marcel Schmidt
Energy forms (I)

March 19th, Sunday

09:25–10:10 Xiang Dong Li
Harnack inequalities and entropy formulae on manifolds with $(K, m)$-super Ricci flows

10:20–11:05 Gilles Carron
Some geometric inequalities induced by the Ricci curvature in the Kato Class

11:15–12:00 Michael Hinz
Essential self-adjointness of Laplacians and two-parameter processes

13:20–14:50 Masatoshi Fukushima
Reflections at infinity of time changed RBMs on a domain with Liouville branches

15:20–16:50 Marcel Schmidt
Energy forms (II)

March 20th, Monday

09:25–10:10 Shin-ichi Ohta
Rigidity for the spectral gap inequality on RCD$(K, \infty)$-spaces

10:20–11:05 Kazumasa Kuwada
Monotonicity and rigidity of the W-entropy on RCD (0,N) spaces

11:15–12:00 Kazuhiro Kuwae
Radial processes on RCD-spaces

12:00– Free time

18:00– Banquet

March 21st, Tuesday

09:25–10:10 Atsushi Kasue
Convergence of Dirichlet forms induced on boundaries of nonparabolic weighted Riemannian manifolds

10:20–11:05 Satoshi Ishiwata
Heat kernel estimates on manifolds with ends and their applications

11:15–12:00 Minoru Murata
Martin boundaries of the products of Riemannian manifolds

13:20–14:05 Guan-Yu Chen
Products of random walks on finite groups

14:15–15:00 Yuichi Shiozawa
Some topics related to global properties of Markov processes

15:20–16:50 Marcel Schmidt
Energy forms (III)

March 22nd, Wednesday

09:25–10:10 Bobo Hua
The Davies-Gaffney-Grigor'yan estimate on graphs

10:20–11:05 Yoshihiro Tawara
Compactness of Markov and Schrödinger semigroups: A probabilistic approach

11:15–12:00 Masayoshi Takeda
Criticality and Subcriticality for Positive Schrödinger Forms

13:20–14:05 Kazuhiro Ishige
The heat kernel of a Schrödinger operator with inverse square potential

14:15–15:00 Radoslaw Wojciechowski
The Feller property and uniform transience

15:20–16:50 Short Communications

March 23rd, Thursday

09:25–10:10 Naotaka Kajino
Weyl's eigenvalue asymptotics for the Laplacian on the Apollonian gasket and on circle packing limit sets of certain Kleinian groups

10:20–11:05 Jun Kigami
Completely Symmetric Resistance Forms on the Stretched Sierpinski Gasket

11:15–12:00 Masanori Hino
An integrated version of Varadhan's asymptotics for lower-order perturbations of strong local Dirichlet forms

13:20–14:05 Jian Wang
Intrinsic Ultracontractivity of Symmetric Pure-Jump Processes on Unbounded Domains

14:15–15:00 Takashi Kumagai
Time changes of stochastic processes associated with resistance forms

This workshop is partially supported by JSPS Program for Advancing Strategic International Networks to Accelerate the Circulation of Talented Researchers and Grant-in-Aid for Scientific Research (B) (PI: N. Obata).

## 1 Masatoshi Fukushima

Title 1: Liouville property of harmonic functions of finite energy for Dirichlet forms

Abstract: A quasi-regular Dirichlet form is said to have a Liouville property if any associated harmonic function of finite energy is constant. We first examine this property for the energy form $\mathcal{E}^\rho$ on $R^n$ generated by a positive function $\rho$. We next make a general consideration on a regular, strongly local and transient Dirichlet form $\mathcal{E}$ and an associated time changed symmetric diffusion process $\check{X}$ with finite lifetime. We show that $\check{X}$ always admits its one-point reflection $\check{X}^*$ at infinity by constructing the corresponding regular Dirichlet form. We then prove that, if $\mathcal{E}$ satisfies the Liouville property, a symmetric conservative diffusion extension $Y$ of $\check{X}$ is unique up to a quasi-homeomorphism, and in fact, a quasi-homeomorphic image of $Y$ equals the one-point reflection $\check{X}^*$ of $\check{X}$ at infinity.

Title 2: Reflections at infinity of time changed RBMs on a domain with Liouville branches

Abstract: Let $Z$ be the transient reflecting Brownian motion on the closure of an unbounded domain $D \subset R^d$ with $N$ number of Liouville branches. We consider a diffuion $X$ on $\overline{D}$ having finite lifetime obtained from $Z$ by a time change. We show that $X$ admits only a finite number of possible symmetric conservative diffusion extensions $Y$ beyond its lifetime characterized by possible partitions of the collection of $N$ ends and we identify the family of the extended Dirichlet spaces of all $Y$ (which are independent of time change used) as subspaces of the space $\mathrm{BL}(D)$ spanned by the extended Sobolev space $H^1_e(D)$ and the approaching probabilities of $Z$ to the ends of Liouville branches.

## 2 Marcel Schmidt

Title: Energy forms

Abstract: In this series of talks we present energy forms, i.e., lower semicontinuous Markovian quadratic forms on the topological vector space $L^0(m)$, where $m$ is a localizable measure, and discuss their global properties.

Energy forms provide a common framework for the study of Dirichlet forms (perturbed by the $L^2$-norm), extended Dirichlet forms and resistance forms. Since we do not assume any smoothness of the underlying space and only a very weak finiteness assumption on the measure, classical tools such as resolvent approximations or the Beurling-Deny representation are not available for energy forms. Instead, we use form methods that are more algebraic in nature to investigate recurrence and transience, Silverstein uniqueness and a conservation property, which turns out to be equivalent to stochastic completeness in the Dirichlet form case.

More precisely, we explain how the algebraic and the order properties of energy forms can be employed to construct reflected energy forms (the maximal Silverstein extension), to introduce weak solutions to the associated Laplace equation and to define a potential operator. We discuss how these notions, which generalize the classical notions that are available for (quasi-)regular Dirichlet forms, allow us to prove several well-known meta theorems of Dirichlet form theory for all energy forms. In particular, we elaborate on the relation of global properties and weak (super-)solutions to the Laplace equation.

## 3 Takashi Shioya

Title: Compactification of the space of metric measure spaces

Abstract: Gromov introduced a geometric theory of metric measure spaces without detailed proofs. In this talk, I will explain a part of his theory, containing the compactification of the space of metric measure spaces and a compactness theorem for compact Riemannian manifolds under a condition of spectrum of Laplacian.

## 4 Asuka Takatsu

Title: High-dimensional metric-measure limit of Stiefel and Grassmann manifolds

Abstract: A metric measure space is the triple of a complete separable metric space with a Borel measure on the space. Gromov defined a concept of convergence of metric measure spaces by the convergence of the sets of 1-Lipschitz functions on the spaces. We study the high-dimensional limit of Stiefel and Grassmann manifolds in the sense of this convergence; the limits are either the infinite-dimensional Gaussian space or its quotient by an mm-isomorphic group action, which are drastically different from the manifolds. This is a joint work with Takashi SHIOYA (Tohoku University).

## 5 Melchior Wirth

Title: Geometry of Dirichlet forms under order isomorphisms

Abstract: Kac famous question 'Can one hear the shape of a drum' asks whether two domains with unitarily equivalent Laplacians are necessarily isometric. By now it is known that the answer is no in general. Following Arendt, we pursue a different approach replacing the unitary intertwiner by an order isomorphism and study the analogous question for generators of Dirichlet forms. In this context we show that the intertwining order isomorphism is unitary (up to a constant) if the Dirichlet forms are irreducible. We obtain transformation formulae for the components in the Beurling-Deny decomposition and condition under which the order isomorphism induces an isometry with respect to suitable metrics associated with the Dirichlet forms. For the latter, recurrence play a crucial role in the non-local case. This is joint work with Matthias Keller, Daniel Lenz, and Marcel Schmidt.

## 6 Xiang Dong Li

Title: Harnack inequalities and entropy formulae on manifolds with $(K, m)$-super Ricci flows

Abstract: In this talk, we present the Li-Yau type Harnack inequality and Hamilton type Harnack inequality for the heat equation associated with the time dependent Witten Laplacian on complete Riemannian manifolds equipped with a variant of the $(K, m)$-super Ricci flows and the $(K, \infty)$-super Perelman Ricci flows. Moreover, we present the

$W$-entropy formulae for the heat equation on manifolds with $(K, m)$-super Ricci flows. Joint work with Songzi Li.

## 7 Gilles Carron

Title: Some geometric inequalities induced by the Ricci curvature in the Kato Class

Abstract: It is now well known that a lower bound on the Ricci curvature yields geometric and analytic estimates on the volume of balls, heat kernel, Poincaré inequality$\cdots$We will discuss on the possibility that some of these properties are induced by spectral properties of the Schrödinger operator whose potential is a multiple of the lowest eigenvalue of the Ricci tensor.

## 8 Michael Hinz

Title: Essential self-adjointness of Laplacians and two-parameter processes

Abstract: It is well known that if one removes a closed set from a complete Riemannian manifold, the Markov uniqueness of the Laplacian with smooth functions, compactly supported in the complement of that set, can be characterized using 1-capacities and hitting probabilities of Brownian motion. We study analogous characterizations for essential self-adjointness, then in terms of 2-capacities and, in cases we can treat, two-parameter processes.

## 9 Shin-ichi Ohta

Title: Rigidity for the spectral gap inequality on RCD$(K, \infty)$-spaces

Abstract:We consider a rigidity problem for the spectral gap of Laplacian on an RCD$(K, \infty)$-space (a metric measure space satisfying the Riemannian curvature-dimension condition) for positive $K$. For a weighted Riemannian manifold, Cheng-Zhou (2013) showed that the sharp spectral gap is achieved only when a 1-dimensional Gaussian space is split off. Generalization to RCD-spaces is not straightforward due to the lack of smooth structure. We employ the theory of regular Lagrangian flows recently developed by Ambrosio-Trevisan to overcome this difficulty. This is a joint work in progress with Nicola Gigli, Christian Ketterer and Kazumasa Kuwada.

## 10 Kazumasa Kuwada

Title: Monotonicity and rigidity of the W-entropy on RCD (0,N) spaces

Abstract: Perelman's $\mathcal{W}$-entropy plays a crucial role in his seminal work on Ricci flow. It is well-known by Perelman's entropy formula that the $\mathcal{W}$-entropy is non-increasing in time and a time derivative vanishes if and only if the space is isomorphic to a gradient shrinking Ricci soliton. L. Ni brought the notion of $\mathcal{W}$-entropy to time-homogeneous

Riemannian manifolds, and the corresponding results has been studied in the literature under nonnegative Ricci curvature.

In this talk, we consider the corresponding problem on $\mathsf{RCD}(0,N)$ spaces, i.e. " Riemannian " metric measure spaces with nonnegative Ricci curvature and upper dimension bound by $N$. By following Topping's approach to this problem by optimal transport, we prove the monotonicity of the $\mathcal{W}$-entropy *without* deriving the entropy formula. Moreover, we also show a rigidity of this monotonicity. Unlike the smooth case, some other singular spaces than Euclidean spaces admit a vanishing time derivative of the $\mathcal{W}$-entropy. Our result is new even on a weighted Riemannian manifold in the sense that we require no additional bounded geometry assumption which is used to derive the entropy formula. This is a joint work with Xiang-Dong Li (Chinese Academy of Science).

## 11 Kazuhiro Kuwae

Title: Radial processes on RCD-spaces

Abstract: We provided a stochastic expression of radial processes on $\mathrm{RCD}(K,N)$-spaces for $K \in \mathbb{R}$ and $N \in ]1,\infty[$. based on the Fukushima decomposition for the radial function $r_p$ and a Laplacian comparison for it. Our expression of radial process is described under the law for every starting point except the reference point $p$. Its proof depends on the upper Gaussian estimate for heat kernel. For the expression under the law for all starting points including the reference point, we need a Bishop type condition at the reference point which is satisfied for Riemannian manifolds or Alexandrov spaces. This is an on-going joint work with Kazumasa Kuwada.

## 12 Atsushi Kasue

Title: Convergence of Dirichlet forms induced on boundaries of nonparabolic weighted Riemannian manifolds

Abstract: In this talk, a certain compactification of a connected, nonparabolic weighted Riemannian manifold is studied in connection with the Hilbert space consisting of functions with finite Dirichlet integrals, and it is proved that the Dirichlet form induced on the boundary of a relatively compact domain in the weighted manifold Mosco converges to that on the boundary of the compactification as the domain increases to exhaust the weighted manifold.

## 13 Satoshi Ishiwata

Title: Heat kernel estimates on manifolds with ends and their applications

Abstract: In 2009, Grigor'yan and Saloff-Coste obtained the heat kernel estimates of connected sums of non-parabolic manifolds. In this talk, we prove the heat kernel estimates on connected sums of parabolic manifolds. We also discuss the Poincare inequality on connected sums. This talk is based on a joint work with A. Grigor'yan and L. Saloff-Coste.

## 14 Minoru Murata

Title: Martin boundaries of the products of Riemannian manifolds

Joint work with Tetsuo Tsuchida

Abstract: Determining Martin boundaries of non-compact Riemannian manifolds is an interesting and subtle problem. In this talk we determine the Martin boundary $\partial_M M$ of the product $M = M_1 \times M_2$ of Riemannian manifolds $M_1$ and $M_2$, under the main assumption that the heat kernel $p_2(x_2, y_2, t)$ on $M_2$ is intrinsically ultracontractive, i.e.,there exists a positive decreasing function $C(t)$ on $(0, 1)$ such that

$$p_2(x_2, y_2, t) \leq C(t)\phi_0(x_2)\phi_0(y_2), \quad x_2, y_2 \in M_2,$$

where $\phi_0$ is the normalized positive $L^2$-eigenfunction for the first eigenvalue $\lambda_0$. We show that $\partial_M M$ depends heavily on whether the constant function 1 is small or big as a perturbation of an associated elliptic operator on $M_1$. For constructing $\partial_M M$, we thoroughly exploit parabolic Martin kernels for the heat equation.

## 15 Guan-Yu Chen

Title: Products of random walks on finite groups

Abstract: Concerning the qualitative behavior of Markov chains, mixing times are useful to quantize the convergence to their stationarity, while cutoffs are important in describing the phase transit of mixing times. Since Aldous and Diaconis introduced both concepts in early 1980s, many scholars are engaged in conquering both subjects, including Laurent Saloff-Coste, Yuval Peres and their collaborators. The total variation is the most frequently used measurement of probability distributions but only few frameworks provide efficient bounds on its mixing time and conditions on its cutoff.

In this talk, we consider products of random walks on finite groups with moderate growth and discuss their total variation cutoffs. Through a series of comparison techniques, the total variation cutoff of discrete time Markov chains can be identified with the Hellinger distance cutoff of corresponding continuous time chains. This provides a new scheme to study the total variation mixing of products of random walks. For illustration, we consider a randomized product of random walks on finite cycles. This talk is based on a joint work with Takashi Kumagai.

## 16 Yuichi Shiozawa

Title: Some topics related to global properties of Markov processes

Abstract: In this talk, we are concerned with the spread rate of particles for a branching Brownian motion on the Euclidean space. Here the branching rate is a Kato class measure with compact support. We will show that under some conditions on the branching mechanism and an associated martingale, the linear spread rate is determined by the bottom of the spectrum for the Schrödinger type operator.

In order to prove our assertion, we discuss the exponential growth rate of the expectation of the Feynman-Kac functional with some restriction on the range of the Brownian particle.

## 17 Bobo Hua

Title: The Davies-Gaffney-Grigor'yan estimate on graphs

Abstract: In this talk, we prove the Davies-Gaffney-Grigor'yan estimate for continuous time heat kernels on graphs.

## 18 Yoshihiro Tawara

Title: Compactness of Markov and Schrödinger semigroups: A probabilistic approach

Abstract: It is proved that by Takeda if an irreducible, strong Feller symmetric Markov process possesses a tightness property, then its semi-group is an $L^2$-compact operator. In this paper, applying this fact, we prove probabilistically the compactness of Dirichlet-Laplacians and Schrödinger operators. This is joint work with M. Takeda, K. Tsuchida.

## 19 Masayoshi Takeda

Title: Criticality and Subcriticality for Positive Schrödinger Forms

Abstract: The criticality theory for second-order linear elliptic operators have been developed by Murata, Pinchover. Criticality and subcriticality are extended notions of recurrence and transience of Markov processes. In my talk, we define the criticality and subcriticality of Schrödinger forms through $h$-transform and give an analytic characterization of these properties via the bottom of spectrum of time-changed processes.

## 20 Kazuhiro Ishige

Title: The heat kernel of a Schrödinger operator with inverse square potential

Abstract: We consider the Schrödinger operator $H = -\Delta + V(|x|)$ with radial potential $V$ which may have singularity at 0 and a quadratic decay at infinity. First, we study the structure of positive harmonic functions of $H$ and give their precise behavior. Second, under quite general conditions we prove an upper bound for the correspond heat kernel $p(x, y, t)$ of the type

$$0 < p(x, y, t) \le C\, t^{-\frac{N}{2}} \frac{U(\min\{|x|, \sqrt{t}\})U(\min\{|y|, \sqrt{t}\})}{U(\sqrt{t})^2} \exp\left(-\frac{|x-y|^2}{Ct}\right)$$

for all $x$, $y \in \mathbf{R}^N$ and $t > 0$, where $U$ is a positive harmonic function of $H$. Third, if $U^2$ is an $A_2$ weight on $\mathbf{R}^N$, then we prove a lower bound of a similar type.

This is a joint work with Yoshitsugu Kabeya (Osaka Prefecture University) and El Maati Ouhabaz (Université de Bordeaux).

## 21 Radoslaw Wojciechowski

Title: The Feller property and uniform transience

Abstract: We will discuss some geometric conditions for the Feller property to hold on infinite, weighted graphs. We will also discuss the equivalence of a strong version of this property and uniform transience.

## 22 Naotaka Kajino

Title: Weyl's eigenvalue asymptotics for the Laplacian on the Apollonian gasket and on circle packing limit sets of certain Kleinian groups

Abstract: The purpose of this talk is to present the speaker's recent results on the construction of a "canonical" Laplacian on circle packing fractals invariant under the action of certain Kleinian groups (discrete subgroups of the group of Moebius transformations on the Riemann sphere) and on the asymptotic behavior of its eigenvalues.

The simplest example is the Apollonian gasket, which is constructed from a given ideal triangle (the closed subset of the plane enclosed by mutually tangent three circles) by repeating indefinitely the process of removing the interior of the inner tangent circles of the ideal triangles. On the Apollonian gasket, a "canonical" Laplacian (to be more precise, a "canonical" Dirichlet form) was constructed by Teplyaev (2004) as the unique one with respect to which the coordinate functions on the Apollonian gasket are harmonic. The speaker has recently discovered an explicit expression of this Dirichlet form in terms of the circle packing structure of the gasket, which immediately extends to general circle packing fractals and defines (a candidate of) a "canonical" Laplacian on such fractals.

Then the speaker has further studied this Laplacian on more general circle packing fractals. When the circle packing fractal is the limit set of a certain class of Kleinian group (the smallest non-empty closed subset of the Riemann sphere invariant under the action of the group), some explicit combinatorial structure of the fractal is known, which makes it possible to prove Weyl's asymptotic formula for the eigenvalues of this Laplacian. The asymptotic formula involves the Hausdorff dimension and measure of the fractals and is of the same form as the circle-counting asymptotic formula by Oh and Shah (Invent. Math., 2012).

## 23 Jun Kigami

Title: Completely Symmetric Resistance Forms on the Stretched Sierpinski Gasket

Abstract: The stretched Sierpinski gasket, SSG for short, is the space obtained by replacing every branching point of the Sierpinski gasket by an interval. It has also been called "deformed Sierpinski gasket" or "Hanoi attractor". As a result, it is the closure of a countable union of intervals and one might expect that a diffusion on SSG is essentially a kind of gluing of the Brownian motions on the intervals. In fact, there have been several works in this direction. There still remains, however, "reminiscence" of the Sierpinski gasket in the geometric structure of SSG and the same should therefore be expected for diffusions. This paper shows that this is the case. In this work, we identify all the

completely symmetric resistance forms on SSG. A completely symmetric resistance form is a resistance form whose restriction to every contractive copy of SSG in itself is invariant under all geometrical symmetries of the copy, which constitute the symmetry group of the triangle. We prove that completely symmetric resistance forms on SSG can be sums of the Dirichlet integrals on the intervals with some particular weights, or a linear combination of a resistance form of the former kind and the standard resistance form on the Sierpinski gasket.

## 24 Masanori Hino

Title: An integrated version of Varadhan's asymptotics for lower-order perturbations of strong local Dirichlet forms

Abstract: The studies of J. A. Ramírez, T. Ariyoshi, and the speaker, an integrated version of Varadhan's asymptotics holds for Markovian semigroups associated with arbitrary strong local symmetric Dirichlet forms. In this talk, we consider non-symmetric bilinear forms that are the sum of strong local symmetric Dirichlet forms and lower-order perturbed terms. We give sufficient conditions for the associated semigroups to have the asymptotics of the same type. Proof is purely analytic. This is a joint work with Kouhei Matsuura (Tohoku Univ.).

## 25 Jian Wang

Title: Intrinsic Ultracontractivity of Symmetric Pure-Jump Processes on Unbounded Domains

Abstract: We consider a symmetric pure-jump Markov process $X = (X_t)_{t\geq 0}$ on $R^d$ generated by a non-local Dirichlet form with jumping kernel $J(x, y)$. Let $D \subset R^d$ be an unbounded domain (connected open set), and $X^D = (X_t^D)_{t\geq 0}$ be the killed process of $X$ on exiting $D$. We obtain sufficient conditions for the compactness and the intrinsic ultracontractivity of the Dirichlet Markov semigroup $(P_t^D)_{t\geq 0}$ of $X^D$, and lower bound estimates of the corresponding ground state. Furthermore, when $D$ is a horn-shaped region, we obtain if and only if conditions for intrinsic ultracontractivity, and established sharp explicit two-sided estimates of ground state in terms of the jumping kernel, the reference function corresponding to the horn-shaped region, and the distances to the boundary and the origin.

## 26 Takashi Kumagai

Title: Time changes of stochastic processes associated with resistance forms

Abstract: In recent years, interest in time changes of stochastic processes according to irregular measures has arisen from various sources. Fundamental examples of such time-changed processes include the so-called Fontes-Isopi-Newman (FIN) diffusion, the introduction of which was motivated by the study of the localization and aging properties

of physical spin systems, and the two-dimensional Liouville Brownian motion, which is the diffusion naturally associated with planar Liouville quantum gravity.

This FIN diffusion is known to be the scaling limit of the one-dimensional Bouchaud trap model, and the two-dimensional Liouville Brownian motion is conjectured to be the scaling limit of simple random walks on random planar maps.

We will provide a general framework for studying such time changed processes and their discrete approximations in the case when the underlying stochastic process is strongly recurrent, in the sense that it can be described by a resistance form, as introduced by J. Kigami. In particular, this includes the case of Brownian motion on tree-like spaces and low-dimensional self-similar fractals. If time permits, we also discuss heat kernel estimates for the relevant time-changed processes.

This is a joint work with D. Croydon (Warwick) and B.M. Hambly (Oxford).