Local and Global Hölder Properties of the Density of the Solutions of SDEs with Singular Coefficients

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Abstract

In this presentation, we will discuss about the regularity of the density of SDE under some assumptions. In many cases, we can get some absolute continuity results of the density of SDE with smooth diffusion coefficient under a mild assumption on the drift coefficient: for example, boundedness or linear growth. So the drift does not seem to be important and the density seems to be smooth if the diffusion coefficient is smooth. However, if we discuss the differentiability or Hölder continuity of the density, the drift may be important. For example, in Section 6.5 in Chapter 6 of "Brownian Motion and Stochastic Calculus" by I. Karatzas and S. E. Shreve, the density is not differentiable even though the diffusion coefficient is smooth. In this case, the drift is bounded but not continuous so our primary interest target is a non-smooth drift case.

Consider a uniformly elliptic one dimensional stochastic differential equation with locally smooth diffusion coefficient and Hölder continuous drift and assume that the weak solution exists. We prove that the density of the solution locally exists and is Hölder continuous. In concrete terms, if the diffusion coefficient is infinitely differentiable and the drift is α-Hölder continuous on some interval then for any $\gamma \in (0, \alpha)$the density is γ-Hölder continuous on this set. Hence the drift may be a determining factor in the Hölder continuity of the density.

Also we prove the existence and global Hölder continuity of the density of the weak solution of a d-dimensional SDE with bounded deterministic diffusion coefficient, bounded drift and compound Poisson process. In this case, we assume that the Fourier transform of the drift exists. In the proof, we treat the integrability of the Fourier transform of the drift instead of a Hölder continuity of the drift in the local case.

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