Calculation of the MEMM for geometric Lévy processes and its application to option pricing

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Introduction

It is well known that the MEMM (minimal entropy martingale measure) for geometric Lévy processes, proposed by Fujiwara and Miyahara [5], plays an important role in the mathematical finance. From the practical point of view, it is also important to calculate concretely the MEMM in order to estimate the price of the derivatives. In this paper, we will give some useful algorithms to obtain the MEMM for the typical (geometric) Lévy processes such as compound Poisson, stable process, etc., and apply these methods to the pricing of Europian call option.

1.MEMM

We review briefly the MEMM according to [5]. Suppose that the underlying asset process S_t is given by $S_t = S_0 \exp\{X_t\}$, where S_0 is an initial value and X_t is a Lévy process with the generating triplet $(\sigma^2, \nu(dx), b)$. Then the MEMM exists if there exists a constant θ^* such that

$$\text{(C1)} \int_{\{|x|\le 1\}} e^x e^{\theta^*(e^x-1)}\nu(dx) < \infty,$$

$$\text{(C2)} \quad f(\theta^*) \equiv b + (\frac{1}{2}+\theta^*)\sigma^2 + \int ((e^x-1)e^{\theta^*(e^x-1)} - I_{\{|x|\le 1\}}x)\nu(dx) = r,$$

where r is the return rate of the save asset. Since $f(\theta)$ is monotone increasing and $f(0) \geq r$ due to the reasonable reasons in the financial market, it may be shown that θ^* is nonpositive. In fact, it is known that there exist negative θ^*'s for many practical cases. In this paper, we propose some useful algorithms to calculate θ^* concretely for some typical geometric Lévy processes by using numerical analysis.

2.Numerical analysys

Although it is easy to show the existence of negative θ^*'s, satisfying the above conditions (C1) and (C2) for many typical Lévy processes such as Brownian motion, compound Poisson, stable, etc., it is rather difficult to obtain θ^* concretely associated with the

MEMM, because we have to solve the nonlinear equation. Assume that $\sigma \neq 0$, then in order to approximate solutions of the Eq.(C2), we use the iteration method, i.e. define $\theta_n, n = 1, 2, \cdots$, by the formula:

$$\theta_{n+1} = \frac{r - f(\theta_n) + \sigma^2\theta_n}{\sigma^2}$$

In the case where $\sigma = 0$, we use the Newtonian method to solve the Eq.(C2), i.e. define $\theta_n, n = 1, 2, \cdots$, by the formula:

$$\theta_{n+1} = \theta_n - \frac{f(\theta_n) - r}{f'(\theta_n)}.$$

Our method of algorithm to obtain θ^* has the following two stages. Firstly we transform $f(\theta)$ into suitable form to use the Monte Carlo method and secondly we apply the iteration method or the Newtonian method. We will apply these methods to compound Poisson, VG(Variance Gamma), stable, CGMY(Carr-Geman-Madan-Yor) and NIG(Normal Inverse Gaussian) processes. Finally, using NIKKEI Average Index Weekly Data of 54 weeks, we shall estimate the price of the Europian option in the case where X_t is compound Poisson or stable processes.

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