One dimensional diffusions conditioned to be non-explosive*

Ichiro SHIGEKAWA† (Kyoto University)

We consider one dimensional diffusions conditioned to be non-explosive. Suppose we are given a diffusion process $\{X_t, P_x\}$ on a state space D. Let ζ be its explosion time. If $P_x[\zeta = \infty] > 0$, then the measure conditioned to be non-explosive is defied by

$$P_x[\ \cdot \mid \zeta = \infty] = P_x[\ \cdot\ \cap \zeta = \infty]/P_x[\zeta = \infty].$$

If $P_x[\zeta = \infty] = 0$, then the measure conditioned to be non-explosive is defined as the limit

$$\lim_{T\to\infty} P_x[\ \cdot \mid \zeta > T]. \tag{1}$$

If the limit exists and the limit is a diffufin process, we call it a *surviving diffusion.* We are interested in the following problems:

1. When does the surviving diffusion?
2. Characterizasion of the surviving diffusion.

Since

$$E_x[\ \cdot \mid \zeta > T] = E_x\left[\ \cdot\ \frac{1_{\{\zeta>t\}} P_{X_t}[\zeta > T-t]]}{P_x[\zeta > T]}\right],$$

our problem is reduce to show the existence of the limit

$$M_t = \lim_{T\to\infty} \frac{1_{\{\zeta>t\}} E_{X_t}[\zeta > T-t]}{P_x[\zeta > T]} \tag{2}$$

and to show that (M_t) is a martingale. To do this, we show that there exist a λ-harmonic function φ so that

$$\lim_{T\to\infty} \frac{P_y[\zeta > T-t]}{P_x[\zeta > T]} = \frac{\varphi(y)e^{-\lambda t}}{\varphi(x)}. \tag{3}$$

We consider one dimensional (minimal) diffusions on an interval (l_1, l_2). The problem is divided into sevral cases according to the classification of the boundary. The the surviving diffusion is characterized as a h-transform of the original process by the λ-harmonic function φ.

---

*February 21-24, 2006, "Stochastic Analysis for Markov processes and its applications" at Kansai University

†E-mail: `ichiro@math.kyoto-u.ac.jp` URL: `http://www.math.kyoto-u.ac.jp/~ichiro`