Stopping problems of multiplicative functionals and optimal investment with general transaction costs

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Abstract:

Risk-sensitive variational inequalities (QVIs) for optimal investment with general transaction costs are studied. The QVIs are derived to solve impulse control problems formulated for power utility maximization on infinite time horizon with general transaction costs. The QVI for such kind of problem is of ”ergodic type” in which the pair \((u, l)\) of a function \(u\) and a constant \(l\) is considered to be a solution. The constant determines the value maximizing the growth rate of expected power utility to the investor’s total wealth. An optimal strategy is constructed from the function \(u\). The difficulty in solving the QVI lies in that its related stopping problem is of a multiplicative functional, which enforce us to comprehend how to treat the quadratic growth nonlinear term appearing in the QVI from analytical, or probabilistic view points. Indeed we cannot employ the methods based on monotonicity which were effective in classical cases and need to invent other scheme constructing a solution instead. Besides, we would note, in spite of the name of ”ergodic type”, the underlying diffusion process of the relevant QVI is not ergodic and it may include different features from the one studied by M. Robin in this regard as well.