Thoughts about the transition function of jump-type Markov processes

For a diffusion process we have a good idea on how to understand its transition function in terms of the geometry of the state space. For Brownian motion this is obvious, Aronson's estimates give a first comparison result for more general diffusions, and further investigations suggests to use the Riemannian metric associated with certain elliptic operators generating a diffusion. Finally this idea was extended to classes of hypoelliptic operators generating diffusions and for this sub-Riemannian geometry was introduced.

For jump-type Markov processes we have so far no understanding at all of the transition function. This applies even to Lévy processes where "simple" characteristic exponents $\psi(\xi)$ might lead to quite complicated transition functions or where the transition functions of two processes associated with rather "close" characteristic exponents behave quite different.

In case where the process is associated with a general symbol $q(x,\xi)$ the situation is even worse. Already in the "best" and "simplest" cases we are far away of getting Aronson-type estimates for the transition function nor do we have any geometric understanding of their properties.

In this talk we want to address both problems. In a first part we indicate some possibilities to approximate the transition function of a jump-type process in such a way that one might end up with Aronson-type estimates.

The second part is more programmatic and we will raise awareness to the fact that to every ("nice") Lévy process we can indeed associate a natural metric and this extends to certain jump-type processes comparable with a fixed Lévy process. Further there are clear indications why these metrics should play a crucial role in understanding the behavior of jump-type processes. The key ingredient to our observations is Schoenberg's theorem on the isometric embeddings of metric-spaces into Hilbert spaces.