# On a dichotomy of transience and recurrence and its application to the integrability and extendability

Masatoshi Fukushima (Kansai University)

**Abstract:**

Any Lévy process and a certain class of diffusion processes satisfy the dichotomy that the sample path either wonders out to infinity or hits a compact set infinitely often almost surely.

We first show that this is the case for any Hunt process $X$ and its excessive measure $m$ if $X$ is $m$-irreducible, of no killing inside and satifies the additional lower-semicontinuity of the excessive function in a non-symmetric transient case.

For any absorbed one dimensional diffusion on a half line, it is known that the $\alpha$-order approaching probabilty to 0 is positive and integrable with respect to the canonical speed measure $m$ if and only if 0 is regular,or equivalently, $ds$ and $dm$ is finite on $(0, 1)$ not depending on the path behavior at infinity.

We prove that this is the case for any Hunt process $X$ as above and for the $\alpha$-order hitting probability of a compact set under some extra conditions. The proof uses a reduction to a finite measure case by a time change and by the one point extendability of an absorbed Markov process as has been shown in [1]. Observe that the above mentioned dichotomy is invariant under the time change.

[1] Z.Q. Chen, M. Fukushima and J. Ying, Extending Markov processes in weak duality by Poisson point processes of excursions, Proceedings of the Abel Symposium 2005, Stochastic Analysis and Applications, A Symposium in Honor of Kiyosi Ito, Springer

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