INTEGRABILITY OF EXIT TIMES

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For $d = 2, 3, \dots$ and $0 < \beta < 1$, we define the *parabola–shaped region* in $\mathbf{R}^d$

$$\mathcal{P}_\beta = \{x = (x_1, \tilde{x}) \ : \ x_1 > 0\,,\ \tilde{x} \in \mathbf{R}^{d-1}\,, |\tilde{x}| < x_1^\beta\}.$$

Let $0 < \alpha < 2$. By $\{X_t\}$ we denote the isotropic α-stable $\mathbf{R}^d$-valued Lévy process. Let τ_β be the first exit time of $\{X_t\}$ from $\mathcal{P}_\beta$. Let $p \geq 0$. The main result of [1] is that $E_x \tau_\beta^p < \infty$ for (some, hence for all) $x \in \mathcal{P}_\beta$ if and only if $p < p_0$, where the critical exponent of integrability is

$$p_0 = \frac{(d-1)(1-\beta) + \alpha}{\alpha\beta}.$$

When $\mathcal{P}_\beta$ is replaced by an open cone, situation becomes less explicit, but the corresponding critical exponent of integrability of the first exit time can be expresses in terms of the cone's Martin kernel with pole at infinity. This is the subject of [2].

I will discuss results and methods of both papers. References to the corresponding results for the Brownian motion are in [3, 4].

References

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